$$S_1 = \sum_{n_x=1}^{\infty} e^{-\frac{h^2}{8mkTa^2} n_x^2}$$

$$= \sum_{n_x=1}^{\infty} e^{-\alpha_x n_x^2} \quad \text{where } \alpha_x = \frac{h^2}{8mkTa^2}$$

This summation can be replaced by integration condition applied :-

spacing between two successive energy level will be $<< kT$

~~Integration~~ Interparticle separation $>>$ thermal debroglie wavelength.

$d >> \lambda_d$.

$$d \approx \left(\frac{V}{N}\right)^{1/3}$$

de Broglie wavelength :-

$$\lambda = \frac{h}{p}$$

Average thermal energy :-

$$\langle KE \rangle = \frac{p^2}{2m} = \frac{1}{2} kT$$

$$\Rightarrow \quad p = \sqrt{mkT}$$

then $\boxed{\lambda_d \approx \frac{h}{\sqrt{mkT}}}$ $\Rightarrow$ thermal debroglie wavelength

actual expression of

$$\lambda_d = \frac{h}{\sqrt{2\pi mkT}}$$