TOPIC : Appendix

Page (7-8) Density of states

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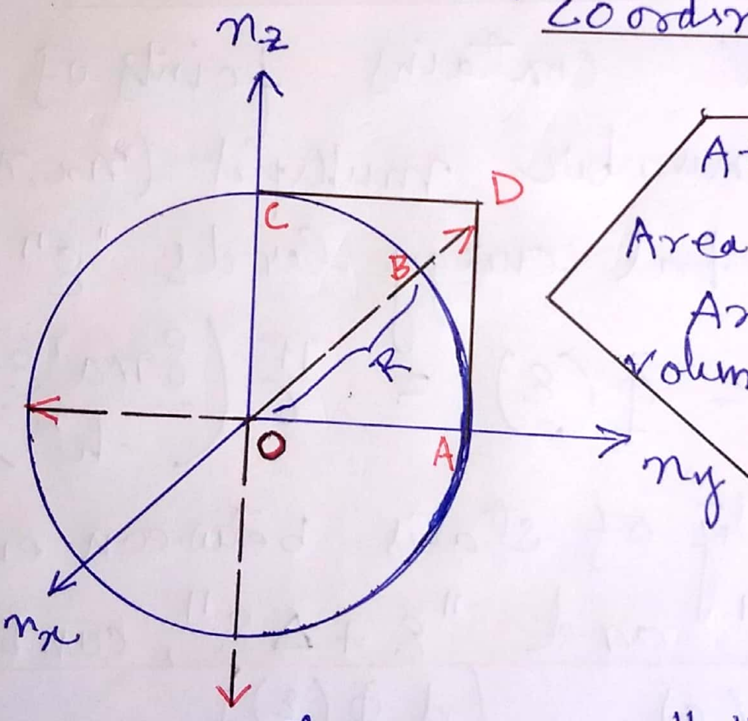
Energy of Particle in 3-D box

$$\mathcal{E} = \mathcal{E}_{n_x, n_y, n_z} = \left(\frac{h^2}{8ma^2}\right)\left(n_x^2 + n_y^2 + n_z^2\right)$$

$$\Rightarrow \left(n_x^2 + n_y^2 + n_z^2\right) = \left(\frac{8ma^2\mathcal{E}}{h^2}\right) = R^2$$

Imagine n_x, n_y & n_z constitutes coordinates



At high values of R
Area (OABC) $\simeq$ Area (OADC)
Volume (OABC part) $\simeq$ Volume (OADC part)

For a certain energy "$\mathcal{E}$", R remains constant

$$\Rightarrow \boxed{\left(n_x^2 + n_y^2 + n_z^2\right) = R^2} \text{ constructs a "sphere"}$$

Since $n_x, n_y, n_z \in \{1, 2, 3, \ldots\}$ all positive integers, all three quantum numbers subtend positive valued quadrant of above Figure, $\Rightarrow \left(\frac{1}{8}\right)$ of full space

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• At a very high values of n_x, n_y & n_z,

"V", volume enclosed by the cube

$\simeq$ volume enclosed by sphere of radius "R"

$$\Rightarrow \boxed{V \equiv \left(\frac{1}{8}\right) \times \left(\frac{4}{3}\pi R^3\right) = \left(\frac{\pi}{6}\right) R^3 \equiv \left(\frac{\pi}{6}\right)\left(\frac{8ma^2\varepsilon}{h^2}\right)^{3/2}}$$

Thus V contains points of all quantum number multiplet (n_x, n_y, n_z) upto total energy levels "ε"

$$\Rightarrow V = \Phi(\varepsilon) = \frac{\pi}{6}\left(\frac{8ma^2\varepsilon}{h^2}\right)^{3/2}$$

Thus Density of states between energy level "ε" and "$\varepsilon + \Delta\varepsilon$", can be defined as

$$\omega(\varepsilon) = \left(\frac{d\,\Phi(\varepsilon)}{d\varepsilon}\right)_{\varepsilon = \varepsilon}$$

$$\Rightarrow \omega(\varepsilon) = \left(\frac{\pi}{4}\right)\left(\frac{8ma^2}{h^2}\right)^{3/2} \varepsilon^{1/2}$$

N.B.: Each point of $(n_x, n_y, n_z) \equiv$ "1" (unit) volume

$\Rightarrow$ unit volume $\equiv$ unit point of quantum number space

$\Rightarrow$ V volume $\cong$ V number of point $\equiv \Phi(\varepsilon)$

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