## **Chapter 18 Radiation Detectors**

A fundamental feature of nuclear processes is that the energy released is larger than the binding energies of atomic electrons. Any emitted particles will have sufficient energy to ionize atoms. Nuclear radiation is called "ionizing radiation", therefore, and detecting this ionization allows us to observe nuclear processes. Radiations that interact with matter via the electromagnetic force, *i.e.*, electrons, charged particles and photons, can directly ionize or excite atoms. These radiations are readily detected. Neutrons interact with nuclei only via the nuclear force and are detected through indirect or secondary ionization processes.

We should note that though the energy released in nuclear processes is several (even many) orders of magnitude larger than atomic binding energies, the total number of ion-pairs that can be created when radiation interacts with matter is small on a macroscopic scale. For example, typical electron binding energies are about 10 eV. If the total energy available from a 1 MeV nuclear decay was completely converted into electron/ion pairs, then the total number of pairs would be ~  $10^5$  corresponding to a charge of ~  $10^{-14}$  Coulombs. Even this estimate of the charge created is optimistic because it is unlikely that all of the energy will create ion pairs. (The "effective" ionization energy of most gases is about 35 eV/ion pair because some ion pairs recombine.)

To measure the radiation, the primary ionization must be preserved and not be lost to recombination or scavenging by electronegative atoms. Metals are not useful for creating radiation detectors, therefore. On the other hand, the created ions must be mobile so that they can be collected. This rules out insulating materials in most cases. The small electrical signals must be amplified to be observed, and so electronic instrumentation plays a role in modern nuclear chemistry.

Although the various types of radiation detectors differ in many respects, several common criteria are used to evaluate the performance of any detector type. The criteria used for this purpose are as follows:

1. The sensitivity of the detector. What types of radiation will the detector detect? For example, solid scintillation detectors are normally not used to detect  $\alpha$ -particles from radioactive decay because the  $\alpha$ -particles cannot penetrate the detector covering.

2. The energy resolution of the detector. Will the detector measure the energy of the radiation striking it, and if so, how precisely does it do this? If two  $\gamma$ -rays of energies 1.10 MeV and 1.15 MeV strike the detector, can it distinguish between them?

*3. The time resolution of the detector or its pulse-resolving time.* How high a counting rate will be measured by the detector without error? How accurately and precisely can one measure the time of arrival of a particle at the detector?

4. The detector efficiency. If 100  $\gamma$ -rays strike a detector, exactly how many will be detected?

Each detector discussed here will be evaluated using these basic criteria.

In this chapter we will consider the techniques developed to detect and quantitatively measure how much ionization and/or excitation is caused by different nuclear radiations. As all radiation creates ionization and/or excitation, we will separate the discussion of detection methods according to the general techniques used to collect and amplify the results of the interaction of the primary radiation with matter rather than by the type of radiation. These detection methods can be classified as: (a) collection of the ionization produced in a gas or solid (b) detection of secondary electronic excitation in a solid or liquid scintillator or (c) detection of specific chemical changes induced in sensitive emulsions.

A brief summary of these detector types is as follows:

#### **Gas Ionization**

Several detector types take advantage of the ionizing effect of radiation on gases. The ion pairs so produced can be separately collected. When a potential gradient is applied between the two electrodes in a gas-filled ion chamber, the positively charged molecules move to the cathode and the negative ions (electrons) move swiftly to the anode, thereby creating a measurable pulse. Such pulses can be readily measured by the associated devices as individual events or integrated current.

#### Ionization in a Solid (Semiconductor Detectors)

In a semiconductor radiation detector, incident radiation interacts with the detector material, a semiconductor such as Si or Ge, to create hole-electron pairs. These holeelectron pairs are collected by charged electrodes with the electrons migrating to the positive electrode and the holes to the negative electrode, thereby creating an electrical pulse. Such pulses contain information on the type, energy, time of arrival, and number of particles arriving per unit time. The important features of semiconductor detectors are their superior energy resolution due to a lower ionization potential and compact size.

#### **Solid Scintillators**

Some of the energy of ionizing radiation can be transferred to fluor molecules (*i.e.*, compounds that can produce fluorescence) in a crystalline solid. The absorbed energy causes excitation of orbital electrons in the fluor. De-excitation causes the emission of the

-3-

absorbed energy as electromagnetic radiation in the visible or near ultraviolet region *(scintillations).* Observing these weak scintillations visually under certain circumstances is possible (see Deeper Look), but visual observation is normally not a feasible detection method. Instead a photomultiplier tube close to the solid fluor is employed. In the photomultiplier, the photons are converted to photoelectrons, which are greatly amplified by secondary electron emission through a series of electrodes (dynodes) to cause a sizable electrical pulse. Thus, the original excitation energy is transformed into a measurable pulse.

#### **Liquid Scintillators**

This detection mechanism is quite similar in principle to the preceding one. Here, however, the radioactive sample and the fluor are the solute in a liquid medium, usually a nonpolar solvent. The energy of nuclear radiation first excites the solvent molecules. This excitation energy eventually appears as photons emitted from the fluor following an intermediate transfer stage. The photons are detected by means of a photomultiplier arrangement.

#### **Nuclear Emulsions**

The process involved here is a chemical one. Ionizing radiation from a sample interacts with the silver halide grains in a photographic emulsion to cause a chemical reaction. Subsequent development of the film produces an image and so permits a semiquantitative estimate of the radiation coming from the sample.

#### **Deeper Look --Visual Detection of Radiation**

There are very few cases in which nuclear radiation can be directly observed by humans. The eye is not sensitive to photons in the x-ray and gamma-ray regions. The energy per decay is small so that only large sources generate enough energy that they warm up and glow. An exception is the Cerenkov radiation emitted by very energetic electrons from the  $\beta$ -decay of fission products and Compton-scattered electrons from  $\gamma$ -decay of these fragments in nuclear reactors. During operation, the reactor produces very neutron-rich fission products that rapidly emit very penetrating  $\beta$ -particles and Compton-scattered electrons. These particles can leave the fuel rods and enter the reactor coolant. The energetic electrons are relativistic and travel with velocities near the speed of light. However, because the speed of light is lower in liquid water than in a vacuum, the electrons emit characteristic blue photons -called Cerenkov radiation- as they adjust their speed

downward. Large power reactors do not have viewing ports, but the cores of research reactors are usually visible. The cores are surrounded by an eerie blue glow when the reactors are operating.



#### **18.1 Detectors Based Upon Ionization**

Many detectors have been developed to collect and amplify the primary ionization created by nuclear particles. In principle, the careful measurement of this ionization provides the most information about the particle and its energy. The devices with the highest resolution are these detectors based upon ionization. Broadly speaking, ionizationbased detectors have the common feature that the incident radiation creates ion pairs in an active volume of the device. An electric field is applied to the active volume to separate the charge pairs and sweep the ions to the electrodes.

Ionization-based detectors have mostly used gases as the active medium. Very few devices use liquids because extremely pure materials are needed to preserve the primary ionization. Gas-filled detectors are easy to construct and operate but the density of the stopping material is low. The effective ionization potential is large, typically ~20 eV. Semiconductors are ~  $10^3$  times denser than gases and have lower "ionization potentials",

 $\sim$  2 eV, but producing large volumes of suitably pure material is expensive. Liquids also have high densities, however, successful devices have only been made with liquefied rare gases, liquid argon and xenon. The impurity level has limited these devices.

#### **18.1.1 Gas Ionization Detectors**

As an energetic charged particle passes through a gas, its electrostatic field will dislodge orbital electrons from atoms sufficiently close to its path. In each case, the negatively charged electron dislodged and the more massive positive ion comprising the remainder of the atom form an *ion pair*. The minimum energy (in electron volts) required for such ion pair formation in a given gas is called the *ionization potential*. This value differs markedly for different gases and is dependent on the type and energy of the charged particle. A more meaningful value is the average energy lost by the particle in producing one ion pair, which is nearly independent of particle energy and type (and is about 35 eV).

The *rate of energy loss* will depend on the energy and type of charged particle. Alpha particles will create intense ionization ( $-10^4$ - $10^5$  ion pairs /cm of path length) whereas  $\beta$ -particles will produce  $10^2$  - $10^3$  ion pairs/cm and the passage of  $\gamma$ -rays will result in 1-10 ion pairs/cm.

#### Ion Chambers

How can we use this primary ionization to produce a detectable signal? The first class of devices to be discussed is the *pulse type ion chambers*. A sketch of such a device (a parallel plate ion chamber) is shown in Figure 18-1. Note that one electrode has been connected to the negative terminal of the voltage source, making it the cathode while the other electrode acts as the anode.

If a 3.5-MeV  $\alpha$ -particle traverses the chamber, intense ionization will occur along its short path. Since about 35 eV are expended, on the average, in forming an ion pair in air, the 3.5-MeV  $\alpha$ -particle could form approximately 1 x 10<sup>5</sup> such ion pairs before dissipating all its energy. Because of the potential on the chamber electrodes, these ions migrate rapidly to the respective electrodes. The less-massive electrons move very quickly to the anode and produce a rapid buildup of charge there. (Because the positive ions move about 1000 times slower than the electrons, their effect can be neglected for the moment.) The time for collection of the electron charge is about 0.1 to 1 µsec depending on the volume of the chamber and the potential gradient. The magnitude of this charge due to the electrons can be calculated as follows:

$$10^{5}e^{-x} 1.6x 10^{-19} Coulomb/e^{-} = 1.6 \times 10^{-14} Coulomb$$
 (18-1)

The collected charge flows through the external circuit as a surge, or pulse. If a 20 picofarad (pF) capacitor is used, the potential of the pulse, V (t), is found as follows:

where Q is the charge (Coulombs) and C is the capacitance (farads). For this case, V=0.0008 volts. The precise measurement of such small pulses is difficult and sensitive low noise electronic modules are needed to measure the signals accurately. Note that in these ion chambers, *there is no amplification of the primary ionization*.

The discussion up to now has not been completely accurate in that the effect of the positive ions on charge collection has been totally neglected. In practice, the positive ions are troublesome. Although they move very slowly to the cathode, as they move, they induce a charge on the negative electrode. If no correction is made for this induced charge, the

size of the output pulse will depend on the position of the particle track in the chamber volume. A simple method for eliminating this positive ion induction is the addition of a grid to the ionization chamber, as shown in Figure 18-2.

An "internal sample" can be placed on the cathode or high-voltage electrode. The grid is charged positively with respect to the cathode, but it is less positive than the anode or collecting electrode. The grid acts to shield the collecting electrode from the effects of the positive ions and accelerates electrons toward the anode.

In many applications, instead of recording pulses from each particle that strikes an ionization chamber, the charge from several events is integrated or added. The total current from the chamber is then measured as a function of time. These devices are generally useful for high radiation field measurements. For example, if one 3.5 MeV  $\alpha$ -particle produces 10<sup>5</sup> ion pairs, if we have 10<sup>7</sup> particles/sec entering the chamber, we will produce 10<sup>12</sup> ion pairs/sec, producing a current of 10<sup>-7</sup> amperes, which can be readily detected.

The electronic signals from the passage of individual particles through an ion chamber can be accurately measured, along with the energy deposited in the gas as a function of position inside the volume. The *rate of ionization is* a characteristic of the nature of the radiation. For example, devices with multiple anodes have been constructed to take samples of the rate of ionization (Figure 18-3). Bragg curve counters determine the relative ionization along the path of the particle by measuring the time distribution of ions as they arrive at the anode. These detectors require sophisticated electronic readout to measure the ionization collected as a function of time while a particle passes through the

gas volume. Therefore, Bragg curve detectors and segmented anode ionization chambers are usually only used to detect charged particles from nuclear reactions.

The most sophisticated gas ionization detector is the *time projection chamber (TPC)*. It is a large, gas-filled cylinder with a negative high voltage electrode at its center. Electrons produced when ionizing radiation passes through the chamber drift towards the end of the cylinder under the influence of axial magnetic and electric fields. The location where the electrons hit the end of the chamber is measured by a set of anode wires. The arrival time of the electrons and the total charge deposited are also recorded. The original particle trajectory through the chamber can be reconstructed from this information.

Gas-filled ionization counters that collect the primary electrons on a wire, as opposed to a plate, can internally amplify the initial ionization. The cylindrical electric field can be very large near thin wire anodes (~  $50\mu$ m), causing the primary electrons to be accelerated past the point at which they create a secondary ionization cascade (Figure 18-4). The secondary ions so formed are accelerated by the prevailing potential gradient, thereby producing still more ionization. Thus, from a few primary ion pairs, a geometrical increase results in a veritable torrent of ions moving toward the chamber electrodes. The process described is known as *gas amplification;* the flood of ions produced is termed the *Townsend avalanche,* in honor of the discoverer of this phenomenon. Because of gas amplification, most of the electrons are collected at the anode within a microsecond or less from the entrance of a single charged particle into the chamber. A strong pulse is thereby formed, fed into the external circuit, and is directly measured after only low amplification.

As the potential gradient between the electrodes of the ionization chamber is further increased, the number of electrons, mostly secondary, reaching the anode rises sharply for a given original ionization event from a charged particle. Eventually a potential will be reached at which the chamber undergoes continuous discharge and is no longer usable as a detector. There are two distinct potential regions between the ion chamber region (no amplification) and continuous discharge that are useful for gas ionization devices. They are called the proportional region and Geiger-Müller region.

#### **Proportional Counters**

In detectors operating in the *proportional region*, the number of ions that form an output pulse is very much greater than, yet proportional to, the number formed by the initial ionization. Gas amplification factors of about  $10^3$ - $10^4$  are generally obtained. The amplification factor is primarily dependent on the composition of the chamber filling gas and the potential gradient. At a given potential, the amplification factor is the same for all ionizing events. Consequently, if an  $\alpha$ -particle traversing the ionization chamber causes  $10^5$  primary ion pairs, with an amplification factor of  $10^3$ , a charge equivalent to  $10^8$  electrons would be collected at the anode. An incident  $\beta$ -particle, on the other hand, producing only  $10^3$  ion pairs, would, after amplification by the factor of  $10^3$ , result in a collected charge equivalent to only  $10^6$  electrons.

As with simple ionization chambers, then, it is possible to differentiate between  $\alpha$  and  $\beta$ -particles in the proportional region based on pulse size. This is one advantage of operating a detector in the region. Because the amplification factor in the proportional region is so heavily dependent on the applied potential, highly stable high-voltage supplies are necessary.

The avalanche of electrons in proportional detectors is collected only on part of the anode wire. Furthermore, only a small fraction of the gas volume of the ionization chamber is involved in the formation of ions. These factors result in a very short *dead time*, that is, the interval during which ion pairs from a previous ionization event are being collected and the chamber is rendered unresponsive to a new ionizing particle. Ionization chambers operating in the proportional region are thus inactivated for only 1 to 2 µsec following each ionization event. Dead times as low as 0.2 to 0.5 µsec can be achieved, but if a proportional counter is used for spectroscopy purposes, the average time between pulses should be ~100 µsec or greater due to the slower operation of the external amplifiers and other electronics.

Some practical designs for proportional counters are shown in Figure 18-5. In the cylindrical detector, a very thin window of split mica or Mylar plastic covers one end of the tube. It can be so thin (down to 150  $\mu$ g/cm<sup>3</sup>) that the absorption of  $\alpha$ -particles by the window is not extensive. An even more efficient arrangement is found with the hemispherical detector, where the radioactive sample can be introduced directly into the detector chamber. In the hemispherical detector, one detects ~ 50% of all the particles emitted by the source. Such windowless detectors are widely used for  $\alpha$ - and weak  $\beta$ -particle counting.

With either ultra thin end-window or windowless detectors, a certain amount of air leaks into the counting chamber. Both the oxygen and the water vapor of the air reduce the detection efficiency. Detectors of this variety, therefore, must be purged with an appropriate counting gas before counting is started and must be continually flushed at a lower flow rate during the counting operation. Consequently, such chambers are often

-12-

called *gas flow detectors*. The operating potential of the chamber is determined, largely, by the gases used for this purpose. Argon, methane, a 90% argon-10% methane mixture known as P-10 gas, or a 4% isobutane-96% helium mixture known as Q-gas are some commonly used counting gases.

Other high purity gases and gas mixtures are used to fill the detectors. Often argon is used for its relatively high density but fluorocarbons like  $CF_4$  and  $C_2F_6$  and hydrocarbons like isooctane and isobutane are also used in devices designed to detect charged particles.

The electronic instrumentation necessary for the operation of the proportional counter is shown in Figure 18-6. Pulses from the detector pass through a preamplifier and amplifier, where they are shaped and amplified. Emerging from the amplifier, the pulses go to a discriminator. The discriminator is set so as not to trip on noise pulses but rather to trip on radiation pulses of any larger size. The number of discriminator pulses produced is recorded by the scaler.

When the count rate of a sample emitting both  $\alpha$  and  $\beta$ -particles is determined over the voltage range of a proportional detector and the data are plotted, the results are as seen in Figure 18-7. The *characteristic curve* for a proportional detector exhibits two plateaus. The plateau at the lower voltage represents  $\alpha$ -radiation alone because, at this potential range, only the  $\alpha$ -particles, with their much greater specific ionization, produce pulses large enough to trigger the discriminator. Not only may the  $\alpha$ -particles thus be counted separately from accompanying  $\beta$ -radiation at this potential, but also the background radiation counting rate (primarily cosmic rays and  $\gamma$ -rays) is extremely low, on the order of a few counts per hour, as well. As the potential gradient in the chamber is increased, the amplification factor becomes correspondingly greater. Eventually the primary ions produced by even the most energetic  $\beta$ -particles are amplified sufficiently to produce pulses large enough to be recorded. This point represents the *beta threshold*. Further increases in potential gradient allow even the pulses from the weaker beta particles to be registered. The *beta plateau* in the operating voltage has now been reached. The count rate here actually represents alpha plus beta radiation. A good proportional counter has a beta plateau slope of less than 0.2% per 100 volts. The efficiency of proportional detectors for gamma radiation is so low that they are seldom used for gamma counting. Often, in discussions of the proportional counter, one forgets to mention that the proportional counter is an excellent spectrometer (*i.e.*, an energy measuring instrument) for low-energy radiation, such as x-rays.

At still higher potential gradients, the gas amplification factor may reach  $10^8$ . Now even a weak  $\beta$ -particle or  $\gamma$ -ray can create sufficient ion pairs to fill completely the available "ion space" in the chamber. Consequently, the size of the charge collected on the anode no longer depends on the number of primary ions produced, and thus, it is no longer possible to distinguish between the various types of radiation. This potential level is called the *Geiger-Müller region*, after the German physicists who first investigated it. Ionization chambers operated in this potential region are commonly called Geiger-Müller (G-M) detectors. Since the maximal gas amplification is realized in this region, the size of the output pulse from the detector will remain the same over a considerable voltage range until continuous discharge occurs. This fact makes it possible to use a less-expensive highvoltage supply than that required for proportional detectors. Use of a very high amplification factor in the Geiger region is not without problems. One is the longer dead time of the chamber. Following the passage of an ionizing particle through a detector, an electron avalanche occurs along the entire anode wire, resulting in a cylindrical sheath of positive ions around the anode. The number of such positive ions per pulse will be one or more orders of magnitude greater than that in chambers operated in the proportional region. To be neutralized, the positive ions must migrate to the cathode wall. Being much more massive than the electrons, these ions move at a slower velocity in the electrical field. During this migration, the chamber is unresponsive to any new ionizing particles passing through it. Thus the dead time of a detector operated in the Geiger region is from 100 to 300 µsec or more.

A correction for "dead time loss" can be made from the value of the dead-time,  $\tau$ . If the true counting rate is n ( $\tau$ =0), and m is the measured rate we have

$$n=m/(1-m\tau)$$
 (18-3)

So, if we measure a counting rate of 1000 cps with a dead time of  $250\mu$ sec, the true counting rate is

with the measured counting rate being low by 33%.

Another important problem is the perpetuated chamber ionization resulting from complications associated with the discharge of positive ions at the cathode wall of a detector. As a result, we need a means of terminating or *quenching* the perpetual ionization in the detector. This is done by introducing polyatomic organic compounds or halogen gases into the counter gas. These molecules dissociate into neutral species, which stops the discharge. Since they are destroyed during quenching, the lifetime of a G-M tube is typically limited to 10<sup>8</sup>-10<sup>9</sup> pulses.

One of us (GTS), as a graduate student in 1936, was involved with the discovery of the quenching phenomenon in Geiger-Müller counters. He and a fellow graduate student, David C. Grahame, were plagued with erratic Geiger-Müller counter behavior until they discovered the beneficial effect of water vapor, which was introduced by accident into the argon gas in their counter. They found that reliable behavior also followed the admixture of small amounts of other gases, such as ammonia and natural gas. They didn't publish a description of this discovery and were quite interested to read in 1937 the publication by Trost (Z. Phys. <u>105</u>, 399 (1937)) of his observation that ethyl alcohol had a similar quenching effect leading to reliable operations of such counters.

#### **18.1.2 Semiconductor Detectors (Solid State Ionization Chambers)**

As mentioned at the beginning of this section, the primary ionization must be collected to make a direct measurement of the energy of nuclear radiation. Condensed phases have higher densities than gases and so provide more efficient stopping of the radiation per unit length. However, metals allow rapid recombination of the electron/positive ion pairs and insulators inhibit the collection of the charge. Therefore, only semiconductors have been used extensively for radiation detectors. Metals and insulators (like concrete) are used extensively in radiation shielding and some transparent inorganic crystals have a special sensitivity to radiation that is discussed below.

Silicon and germanium are the most common semiconductors used to construct "solid state ionization chambers." These materials must be extremely pure to observe the primary ionization ( $\sim 10^5$  electrons) and, as we will see below, germanium devices must be

cooled to reduce the thermal noise to observe the signals. The properties of small-scale devices based on Group III/Group V materials, e.g., GaAs, have been studied but no large-scale applications have been made. The size and shape of the available semiconductors have grown over time but are still severely limited by production techniques and the availability of high purity material.

Early solid-state devices relied on observing the ionization in intrinsic semiconductors. Early devices were impractical due to the requirement of extremely pure material. Modern devices are based on semiconductor junction diodes. These diodes have a rectifying junction that only allows the flow of current in one direction. Incident radiation creates ionization inside the bulk of the diode, and creates a pulse of current in the opposite direction to the normal current flow through a diode that is straightforward to detect.

To understand how semiconductor radiation detectors operate, it is necessary to review a little of the basic chemistry of semiconductors. Consider a typical Group IV element, such as Si or Ge. It will crystallize in the diamond lattice structure, as shown in Figure 18-8. Each silicon atom is bound by four electron-pair bonds to adjacent silicon atoms. The electrons are not free to migrate through the crystal, and therefore pure silicon is a poor conductor of electricity. The electron energy levels of silicon are shown in Figure 18-9. The electron energy levels of the valence electrons are so close together that they form a nearly continuous "band" of energies, known as the *valence band*. In pure silicon, there is a region of energies above the valence band in which there are no allowed energy levels. This energy region is called the *forbidden gap* and corresponds to  $\sim$ 1.08 eV for silicon. Just above the forbidden gap is the *conduction band*, another band of energies that

allows free electron migration through the crystal, *i.e.*, the conduction of electricity. Suppose we replace a silicon atom in the silicon lattice with a Group V atom, such as phosphorus. Then we will have the situation depicted in Figure 18-10. Phosphorus has five valence electrons. After forming four electron pair bonds to the adjacent silicon atoms, there is one electron left over. This leftover electron will be very loosely bound to the phosphorus atom and will be easily removed to conduct electricity through the crystal.

In terms of our diagrams of the crystalline-electron-energy levels, we have the situation shown in Figure 18-11. The "extra" phosphorus electron occupies a "donor level" very close to the conduction band and is easily promoted into this conduction band. Silicon containing Group V impurities, such as phosphorus, is called *n*-type silicon because the species that carries charge through the crystal is negative.

What happens when a Group III atom, like boron, replaces an atom in the silicon lattice? The situation is shown in Figure 18-12. Boron has three valence electrons and can form electron pair bonds with three of its neighbors. It has no electron to pair up with the electron on the fourth silicon atom. We say we have an electron *hole* in the silicon lattice.

In terms of our energy level diagrams, we have the situation illustrated in Figure 18-13. The hole occupies an energy level very close to the valence band (*an acceptor level*) and can be easily promoted into the valence band. (Promotion of a hole into the valence band simply means that an electron in the valence band and a hole in the acceptor level switch places, so that a hole is created in the valence band.)

We must realize (unphysical as it may sound) that a hole in the valence band, can conduct electricity as well as an electron in the conduction band. How does this work? Consider Figure 18-14. Imagine that electron 1 moves to fill hole 0. This step creates a hole at position 1. Electron 2 moves to fill this hole, leaving a hole at position 2. Electron 3 fills the hole at position 2, leaving a hole at position 3, and so forth. Thus as the hole moves to the right in Figure 18-14, negative charge is moving toward the left. Since electricity is the movement of charge, the motion of the hole corresponds to the flow of electricity. Silicon containing Group III impurities is said to be *p-type silicon* because of the positive charge carriers (the holes).

A silicon-semiconductor-radiation detector of a layer of p-type silicon in contact with a layer of n-type Si is shown in Figure 18-15. What happens when this *p-n junction* is created? The electrons from the n-type silicon will migrate across the junction and fill the holes in the p-type silicon to create an area around the p-n junction in which there is no excess of holes or electrons. (We say that a *"depletion region"* has been formed around the junction.) Imagine that we apply a positive voltage to the n-type material and a negative voltage to the p-type material (the junction is said to be *reverse-biased*). The electrons will be "pulled farther away" from the junction by the positive voltage on the n-type material, thus creating a much thicker depletion region around the p-n junction. The exact thickness of the depletion region, d, is given by

$$\mathbf{d} \propto (\rho \mathbf{V})^2 \tag{18-4}$$

where  $\rho$  is the resistivity of the silicon and V is the magnitude of the applied reverse-bias voltage. Note that the depletion of the depletion region can be varied at will by changing the voltage applied to the detector.

The depletion region acts as the sensitive volume of the detector. The passage of ionizing radiation through this region will create holes in the valence band and electrons in the conduction band. The electrons will migrate to the positive charge on the n side, while

the holes will migrate to the negative voltage on the p side, thereby creating an electrical pulse at the output of the device.

The *average* energy necessary to create a hole-electron pair in silicon is  $\sim 3.6$  eV. [This average energy is about three times the forbidden gap energy ( $\sim 1.1 \text{ eV}$ ) because most electrons are promoted from deep in the valence band to high in the conduction band.] The energy required to create a hole-electron pair is independent of particle charge and mass, thus causing semiconductor detector response to be independent of particle type. If we remember that the average energy to create an ion-electron pair in a gas ionization device was  $\sim$ 35 eV, then we see that, for the same energy deposit in the detector, we get  $\sim$  35/3.6 ≈10 times more charged pairs. If we note that the energy resolution of a detector,  $\Delta E/E$ , is proportional to  $N^{-1/2}$  where *N* is the number of charge pairs formed, we can see that the energy resolution of a semiconductor is approximately  $10^{1/2} = 3.2$  times better than the energy resolution of a gas ionization detector. (Furthermore, as we will see later the average  $\gamma$ -ray energy deposit required to liberate one photoelectron at the cathode of a photomultiplier tube is  $\approx 1000$  eV, then we can say that the resolution of a semiconductor detector is  $(1000/3.6)^2 \approx 17$  times better than that of a scintillation detector.) More detailed considerations show that the observed resolution can be smaller than the estimate based on the statistics of ion pair formation due to correlations between processes giving rise to ion pair formation

For some semiconductor detectors, germanium is used instead of silicon for the detector material. The reasons for this substitution are as follows: (a) The average energy needed to create a hole-electron pair in germanium is 2.9 eV rather than the 3.6 eV necessary for Si. Thus the energy resolution for germanium should be  $(3.6/2.9)^2=1.1$  times

better than silicon. (b) The atomic number of germanium (32) is much higher than that of silicon (14), leading to increased probability of  $\gamma$ -ray interaction with the detector material. Consequently, germanium is preferred to silicon for  $\gamma$ -ray detection. The forbidden gap is so small, however, for germanium (0.66 eV) that room temperature thermal excitation leads to the formation of hole-electron pairs in the solid. Therefore germanium detectors must be operated at liquid nitrogen temperature (77°K) to prevent this thermal electron noise from overwhelming the small signals from the primary ionization.

The silicon-based solid-state detectors fall into three general categories, surface barrier devices, PIN diodes, and Si (Li)(pronounced "silly") devices. These detectors are used to measure short-ranged radiation: charged particles in the first two cases and low energy gamma rays and x-rays in the third case. The detector consists of a thin layer of silicon material, (often ~ 200 $\mu$ m thick, but thicknesses from five  $\mu$ m up to 5 mm are available). An electric field (typically ~V/ $\mu$ m) is applied in the direction opposite to the "normal" flow of current through the diode. Radiation creates electron/hole pairs that are swept to the electrodes by the electric field and induce a current signal. These signals are amplified in an external circuit.

*Silicon-surface barrier* detectors *(SSB)* consist of a thin cylindrical piece of high resistivity (10<sup>3</sup> ohm cm) pure n-type silicon with a thin gold contact on one side and an aluminum contact on the other. The gold contact is a thin layer through which the radiation enters the silicon. Just under the gold is an oxide layer that forms the semiconductor junction (or barrier). The gold layer is sensitive to physical wear and the oxide layer can be depleted by extended exposure to vacuum. The oxide layer is also very sensitive to organic molecules but can be reconstructed with proper treatment. During use these detectors

-21-

must be shielded from visible light as electron/hole pairs can be created by photons that enter the silicon through the thin gold contact. The gold and oxide layers are also thin to reduce the amount of kinetic energy lost by the particle before it enters the active silicon region. These layers make up a dead-layer that can be significant in alpha-spectroscopy. Recently, SSB's with very thin and uniform dead-layers have been created with ionimplanted junctions. A thin layer of boron ions is implanted near the surface of n-type silicon to form the junction.

"Ruggedized" detectors are available in which the radiation enters the silicon through the thicker and light-tight aluminum contact. The bulk of the material is p-type silicon and a negative bias is applied to the gold contact so that the entry window can remain at ground potential. Another design of a "rugged" surface barrier detector replaces the gold contact with a thicker nickel contact. The nickel is resilient enough to be wiped clean.

Silicon surface barrier detectors have found wide spread application in alpha particle spectroscopy and in nuclear reaction studies. These detectors can be used in stacks to identify particles uniquely. Consider the situation in which a penetrating ion with a total kinetic energy KE passes through a thin SSB detector and is stopped in a second thicker SSB detector. Such a stack of detectors is called a silicon detector telescope and provides two signals, the energy lost in the thinner detector,  $\Delta E$ , and the remainder, KE- $\Delta E$ . The rate of energy loss for a charged particle is approximately given by the simple expression:

$$dE/dX \sim mZ^2/KE \tag{18-5}$$

-22-

where m is the ion's mass, Z its nuclear charge, for a given kinetic energy. When the first detector is thin, dE/dX is approximately constant and  $\Delta E \approx (dE/dX) \Delta X$ . Thus,  $\Delta E$  will be a hyperbolic function of KE for each ion with a different value of mZ<sup>2</sup>. Therefore, the components of a mixture of penetrating charged particles can be identified by their relative values of ionization. Stacks of SSB's with several thin detectors are used when unambiguous particle identification through redundant measurements is necessary. Variations in the thickness of the silicon in the manufacturing process place a practical limit on the particle identification. Thickness variations as small as 1 µm are currently achievable.

The typical resolution of a single detector is ~20 keV but depends on the detector geometry, in particular, on the detector capacitance. Notice that SSB detectors have parallel electrodes separated by a thin dielectric; the capacitance of such an object will increase with increasing area and with decreasing thickness. Thus, thin large area devices will have the largest capacity and thus the poorest resolution.

Silicon PIN diodes are a more recent class of detectors that have become available, in large part, due to the growth of the semiconductor industry. These devices are made up with a p-type layer on one side of an intrinsic silicon wafer and an n-type layer on the opposite, therefore a p-I-n sandwich. The detectors are available in a much larger range of sizes and shapes than surface-barrier detectors. For example, 25 cm<sup>2</sup> devices with single or multiple specially shaped contacts are routinely available. The contacts on the front and back of PIN diodes can have different shapes and sizes. Consider a 5cm x 5cm rectangular wafer that has two horizontal electrical contacts on the front and two vertical contacts on the back. The divided contacts define four quadrants of silicon that are electrically separate. A particle that enters the detector will generate two signals, one on the front and another on the back that uniquely identifies the quadrant of silicon. Rectangular devices are often used with sixteen stripes on the front and sixteen stripes on the back and provide very accurate position measurements in nuclear reaction studies.

All solid-state detectors are damaged by long exposures to charged particles. The threshold dose for observable damage (in particles/cm<sup>2</sup>) is ~10<sup>8</sup> for fission fragments, 10<sup>9</sup> for  $\alpha$ -particles, 10<sup>12</sup> for fast neutrons and 10<sup>13</sup> for electrons. Notice that the particles generally come to rest in the silicon and stay there. The lattice will be disrupted and poisoned by the presence of many stopped particles. If the particles have the same energy (same range), then all of the particles will stop in a very narrow band of the silicon and can create a dead layer inside the silicon.

Lithium Drifted Silicon Detectors have been developed for measurements of beta particles. Electrons are more penetrating than heavier charged particles and so these devices, usually called Si (Li) detectors have to be much thicker than heavy-ion detectors. Si (Li) detectors are commonly 5 mm thick. Such large volumes of very pure silicon are not readily available so the technique of drifting lithium ions into the bulk material to compensate for internal lattice defects in p-type silicon has been developed. A layer of lithium metal is applied to the surface and some atoms diffuse into the bulk silicon. The lithium atoms readily donate an electron into the conduction band and become ions. A bias can be applied to the silicon that causes the lithium ions to migrate from the surface through the lattice. The migrating ions will be trapped by negative impurities in the lattice, thus "compensating" for the effect of the impurity. The lithium ions retain their high mobility in the lattice and the detectors have to be stored with a small retaining bias if they are stored for long periods at room temperature.

Si (Li) detectors are favored over Ge (Li) detectors for  $\beta$  detection because of their low  $\gamma$ -sensitivity and their lower (by ~1/3 to ~1/2) backscattering. The energy resolution of Si (Li) detectors for electrons is ~ 1 to 2 keV for electron energies up to 1000 keV. The detection efficiency of Si (Li) detectors for  $\beta$ - particles ranges from one-half that of a gas counter for a low-energy beta emitter like <sup>14</sup>C to greater than that of a gas counter for an energetic  $\beta$ -emitter such as <sup>32</sup>P. The background of these detectors is exceptionally low, because of their small size for a given stopping power, and they do not require any peripheral gas supply, etc. Very good energy resolution for x-ray detection is possible. A resolution of 180 eV for the 5.9 keV Mn K<sub>\alpha</sub> x-ray have been obtained with Si (Li) detectors whereas the best energy resolution available from a scintillation detector is about 1000 eV.

All of the silicon detectors can be cooled to reduce the thermal noise that produces a background under all the induced signals. The thermal noise is created by random fluctuations that promote an electron across the band gap into the conduction band resulting in an electron/hole pair. The number of promoted electrons will be proportional to a Boltzmann function containing the band gap  $\Delta$  and the temperature, T:

$$N_{\text{thermal}} \propto e^{\Delta/kT}$$
 (18-6)

For practical reasons, silicon detectors are usually cooled from room temperature down to approximately -20°C; cooling below -60°C is not useful because the system noise becomes dominated by the external electronic circuit. Temperatures below -20°C are not used also because the internal physical stresses from differences in the thermal expansion coefficients of the construction materials become important. Specially prepared detectors are recommended before cooling to the lower temperatures.

We can estimate the factor by which the thermal noise will be reduced with the Boltzmann expression:

$$N_{\text{thermal}}(T = -20^{\circ}) / N_{\text{thermal}}(T = +25^{\circ}) \sim e^{253/298} = 2.3$$
 (18-7)

Cooled silicon detectors are particularly useful in experiments in which the measured particles are expected to cause significant damage to the crystal lattice during the experiment. If the detector is not cooled, the thermal noise will dramatically change during the measurement and the detector resolution will decrease with time.

*Germanium Detectors* have the highest resolution of any direct ionization devices. This is due to the small band gap of germanium of 0.73 eV (at 80 K) and effective ionization potential of 2.95 eV that allows the creation of many ion pairs for a given amount of radiation. The band gap is also small enough that the number of electron/hole pairs created by thermal fluctuations causes a very significant electronic noise. The noise is reduced by enclosing the germanium and the first stage of the amplification circuit in a cryostat and cooling both to liquid nitrogen temperature.

Other things being equal, the size of the signals produced in a germanium diode compared with a silicon diode should be larger by the inverse ratio of the effective ionization potentials,  $3.76 / 2.95 \sim 1.27$  (at 80 K). However, the thermal noise will be larger in proportion to a Boltzmann exponential distribution with the band gaps, so the noise in the germanium will be larger by the factor:

$$e^{-\Delta_{Ge}/kT} / e^{-\Delta_{Si}/kT} = e^{1.16/0.73} = 4.9$$
(18-8)

These facts would appear to favor the use of silicon detectors strongly. However, the "stopping power" of matter for photons is much lower that than for charged particles giving photons long penetration depths in all materials. Moreover, the probability of a photoelectric interaction with an atom, which contributes significantly to the absorption of the full energy of photons, increases in proportion to Z<sup>5</sup>. This makes high-Z materials more effective absorbers of photons. Also from the practical standpoint, manufacturing techniques have been developed to produce very high purity germanium crystals that are much larger than silicon crystals.

Germanium detectors are used almost exclusively to detect gamma radiation. Energetic photons can easily penetrate the cryostat and the high resolution of germanium detectors are well suited to the very precise energies of the gamma rays emitted by the deexcitation of nuclear levels. There are two main classes of germanium detectors, those that use lithium compensated material, called Ge(Li) (pronounced like *jelly*), and more recently, intrinsic germanium, also called high purity germanium.

Ge(Li) detectors are similar in principle to the Si(Li) detectors described above. Large ingots (~1 L) of p-type germanium material are prepared in a relatively pure state. Lithium metal is applied to the surface, some atoms diffuse into the lattice, donate their electron and the ions are subsequently drifted through the material. The mobility of lithium ions is much higher in germanium than in silicon. This allows very effective compensation of the impurities in large volumes of germanium but also allows the rapid loss of compensation by the lattice at room temperature. The hallmark of Ge(Li) detectors is that the germanium crystals must be kept at liquid nitrogen temperatures for their entire useful lifetime. If the detectors are allowed to warm up, even for a very short time, the

-27-

lithium compensation is lost to some extent and their high resolution is degraded. The crystals can be redrifted but the manufacturer must do this.

The shape of the germanium crystals is generally cylindrical with the lithium applied to the outer surface and drifted in toward the center. The lithium contact produces an n-type region on the surface of the crystal, the bulk becomes intrinsic though compensation, and a small p-type region is deliberately left un-drifted to produce a P-I-N diode structure. The detector thus has coaxial p- and n-type 'electrodes' and is sometimes called a coax detector. A reverse bias is applied to the germanium diode, as to the silicon detectors, and the small current pulses from the primary ionization events are collected and amplified. The requirement that Ge(Li) detectors must be kept very cold during their entire lifespan has spurred the development of germanium purification techniques. Large volumes of intrinsic germanium material can now be produced in sufficient quantity to produce PIN diode detectors without lithium drifting. These devices can be stored at room temperature and only need be cooled when they are used. The n-type region is prepared by lithium diffusion without drifting and an extremely thin p-type region is prepared by implanting boron ions. The crystals are generally cylindrical with coaxial electrodes. Other shapes, such as thin planar detectors for low energy y-rays and X-rays and crystals with hollow wells with large geometrical efficiency are available.

For  $\gamma$ -ray detection, the detector of choice is the Ge detector because of its higher Z. The most spectacular feature of the Ge detector is the superior energy resolution. An energy resolution of 1.75 keV for the 1332 keV  $\gamma$ -ray of <sup>60</sup>Co is routinely obtained. (compared with the typical 90-100 keV for a 3 x 3 in. NaI(Tl) detectors). What this means in terms of the ability to resolve  $\gamma$ -ray spectra is shown in Figure 18-17.

#### **18.2 Scintillation Detectors**

In a scintillation detector, a fraction of the energy deposited by the primary radiation in the detector is converted to light that, in turn, is converted into an electrical signal. Conceptually, the process can be divided into the scintillation process itself (energy light), the collection and conversion of the light into electrons and the multiplication of the electrons to make a macroscopic signal. We will divide our discussion similarly.

As discussed in Chapter 17, as radiation interacts with matter, it will lose energy by ionizing or exciting matter. As we have seen, only a few materials have the right properties to allow the collection of the primary ionization from nuclear radiation. If the ionization is not preserved and collected, the electron/positive ion pairs are expected to recombine eventually. During this recombination, the energy used to separate the charges will be reemitted to the surroundings, very often as lattice vibrations and heat. Occasionally, a triplet electronic excited state is populated and the energy from such states is released as visible photons. This emission process is well known as atomic or molecular fluorescence and is called *scintillation* when it is caused by exciting radiation. (We will use the term "visible light" loosely in our discussion. The wave lengths of the fluorescent photons from excited electronic states are characteristic of the material and range from UV to red.) These visible, secondary photons can be easily detected and amplified with photomultiplier tubes.

The details of the scintillation process are complicated and depend very much on the molecular structure of the scintillator. In organic crystals, the molecules of the organic solid are excited from their ground states to their electronic excited states (see Figure 18-18). The decay of these states by the emission of photons occurs in about 10<sup>-8</sup> sec (fluorescence). Some of the initial energy absorbed by the molecule is dissipated as lattice vibrations before or after the decay by photon emission. As a result, the crystal will transmit its own fluorescent radiation without absorption.

There are three common types of organic scintillator. The first type is a pure crystalline material, such as anthracene. The second type, *the liquid scintillator*, is the solution of an organic scintillator in an organic liquid, such as a solution of p-terphenyl in toluene ( $\approx$ 3 g solute/L solution). The third type is the solution of an organic scintillator, such as p-terphenyl, in a solid plastic, such as polystyrene.

All these organic scintillators are characterized by short fluorescence lifetimes, *i.e.*, 2-3 ns. This allows their use in high-count rate situations or for fast time measurements. The light output is modest, being 10-50% of that of NaI(Tl) (see below). Because of the low Z of the organic scintillators, they are primarily used in the detection of heavy charged particles or electrons. The plastic scintillators are easily machined into a variety of shapes and/or made as thin films. Energy resolutions of about 10-14% are possible. Liquid scintillators are used to assay low energy  $\beta$ -emitters, like <sup>14</sup>C or <sup>3</sup>H. The radioactive material is dissolved or suspended in the scintillator solution. Another application involves the use of liquid scintillators for large volume (several m<sup>3</sup>) detectors where the liquid scintillator has been "loaded" with a neutron absorbing material, such as gadolinium. The gadolinium captures neutrons producing e<sup>-</sup> and  $\gamma$ -rays that are detected by the scintillator.

The scintillation process in inorganic scintillators differs from that in organic scintillators. Consider the structure of an ionic crystal, as shown in Figure 18-19. When an energetic electron passes through the crystal, it may raise valence electrons from the valence band to the conduction band. The electron vacancy in the valence band resulting

from this ionization is called a *"hole"*, in the valence band. The electron in the conduction band and the hole in the valence band can migrate independently through the crystal.

Alternatively, another process called *excitation* can occur by which a valence band electron is excited to an energy level lower than the conduction band. The electron remains bound to the hole in the valence band. This neutral electron-hole pair is called an *exciton* and it can move through the crystal also. Associated with the exciton is a band of energy levels called the exciton band (see Figure 18-19).

The presence of lattice defects and/or intentionally placed impurities in the alkali halide crystal will cause the formation of local energy levels in the forbidden gap, called traps or *activator centers*. Figure 18-19 shows the energy levels of an alkali halide crystal, including the activator centers and traps. (Atomic thallium is a common activator for alkali halide crystals).

Excitons, holes, and electrons produced by the interaction of radiation with the crystal wander through the crystal until they are trapped at an activator center or trap. Migration of an exciton in a crystal may be thought of as a 6 to 8 eV excited iodide ion, I-\*, transferring its energy to an adjacent stable I ·, which, in turn, becomes excited. Thus energy may be transferred from I<sup>-</sup> to I<sup>-</sup> in the crystal lattice to final capture by either an activator center or crystal impurity. By exciton capture or hole-electron capture, the activator centers are raised from their ground state to an excited state. The de-excitation of this activator center by emission of light occurs in a time about 0.3 µsec. Hence the energy deposited by the radiation in the scintillator is emitted as light by the activator center (TI). The amount of light emitted by the entire crystal is directly proportional to the amount of

energy deposited in the crystal by the incident radiation. The fraction of the deposited energy converted into fluorescence photons is small, about 10%.

Thallium-activated sodium iodide (NaI(Tl)) is the most widely used inorganic scintillator. This material is used extensively to detect  $\gamma$ -rays because it is relatively inexpensive, has a high stopping power for photons, and is rugged and easy to use. The fluorescence light output has a relatively slow decay time of almost 230 ns, limiting the count rate in such detectors. The energy resolution of NaI(Tl) detectors is rarely better than 6% for the 1332 keV  $\gamma$ -ray of <sup>60</sup>Co (as compared to the 0.13% typically seen with Ge detectors). NaI(Tl) detectors are very efficient for detecting  $\gamma$ -radiation (with typical detection efficiencies of 1-10%). The efficiency of a 3-inch diameter right cylinder that is 3 inches long is the reference standard for  $\gamma$ -ray detectors. Other inorganic scintillators of note are bismuth germanate (Bi<sub>4</sub>Ge<sub>3</sub>O<sub>12</sub>, BGO) which is a high Z, high-density material. Its low light output (10-20% that of NaI(Tl)) limits its use to situations where a high efficiency (with poorer resolution) is needed. Barium Fluoride (BaF<sub>2</sub>) is a high Z material with a fast light output ( $\tau$ <1ns) with reduced light output. It is used in situations where its high density, high Z and fast timing are important.

How is the light emitted by the scintillator converted into an electrical signal? To answer this question, let us consider a schematic diagram of a typical scintillation detector (Figure 18-20). The photons of visible light emitted by the activator centers, such as Tl<sup>+</sup> ions in a NaI(TI) crystal, pass through the transparent fluor substance and out through a clear window to impinge on an adjacent photocathode. The typical photocathode is composed of a thin, photosensitive layer (commonly a cesium-antimony alloy) on the inner surface of the end of the photomultiplier tube. Here impinging photons, particularly those having wave lengths between 3000 and 6000 A, are absorbed, with a consequent emission of photoelectrons. The number of photoelectrons ejected is slightly less than, but directly related to, the number of incident photons. Such a burst of photoelectrons resulting from a single  $\gamma$ -ray interaction in the crystal is still far too weak to be registered directly.

Amplification occurs by means of a series of electrodes, called *dynodes*, spaced along the length of the photomultiplier tube (see Figure 18-20). Each dynode is maintained at a higher potential (usually about 50 volts higher) than the preceding one. The photoelectrons emitted from the photocathode are focused by the focusing electrode to hit the first dynode. Striking the dynode surface, they cause the secondary emission of a larger number of electrons. This new burst of electrons is attracted by the potential gradient to the second dynode in the series, where a still larger number of electrons are dislodged. This electron-multiplying process continues at each dynode until at last the collecting anode is struck by  $10^5$  to  $10^6$  electrons for each original photoelectron ejected from the photocathode. Thus the size of the output pulse from the photomultiplier is directly related to the quantity of energy dissipated by the incident  $\gamma$ -ray photon in the fluor.

To present a quantitative example of the energy conversions involved in scintillation detection, we trace the results of the interaction of a single 1.17-MeV  $\gamma$ -ray from <sup>60</sup>Co with a thallium-activated sodium iodide crystal [Nal(TI)]:

1. If 20% of the energy of the  $\gamma$ -ray results in exciton production in a fluor crystal, and if it is assumed that 7 eV is needed to produce an exciton, then approximately 33,000 excitons could result from this  $\gamma$ -ray photon.

-33-

- 2. Assuming that only 10% of the excitation events result in the production of photons of visible light seen by the adjacent photocathode, this would mean that about 3300 photons would reach the photocathode.
- 3. This number of photons striking a photocathode with a conversion efficiency of 10% would eject approximately 330 photoelectrons.
- 4. The successive dynodes of a photomultiplier operated at an overall gain of 10<sup>6</sup> could then amplify this quantity of photoelectrons so that  $\sim 3.3 \times 10^8$  electrons would be collected at the photomultiplier anode, or a charge of  $\sim 5 \times 10^{-11}$  Coulomb.
- 5. This charge could then be transformed by a preamplifier circuit with a capacitance of 30 pF into an output pulse of 1.8 volts. A pulse of this size would be capable of directly triggering a scaler.

<sup>60</sup>Co emits two γ-rays per disintegration. The other γ-ray has an energy of 1.33 MeV. Following the preceding calculations, this 1.33-MeV γ-ray would result in an output pulse of about 2.05 volts from the detector.

#### **18.3** Nuclear Track Detectors

The passage of highly ionizing radiation through an insulating solid leaves a wake of destruction in the material. In covalently bonded materials, the chemical structure of the material along the track can be significantly and permanently changed by the passage of a single energetic ion. Certain polymeric (plastic) materials and the mineral mica (a form of silicon dioxide) are particularly sensitive to such radiation damage. The original radiation damage remains localized on the molecular scale but is not visible without enhancement. However, the track can be expanded by chemical etching from the molecular scale (nanometers) up to the microscopic scale (µm).

Nuclear track detectors are very simple and very efficient detectors of rare events that produce highly ionizing radiation. Carefully prepared and scanned track detectors have been used to identify individual rare decays. The detectors are integrating in that the damage caused by a track is not spontaneously repaired. The drawback to track detectors is that the tracks are small and can only be observed with a microscope. In the past, scanning by eye was extremely labor intensive and prone to error. Modern computercontrolled scanning has improved the speed and reliability of the analysis. Plastic track detectors that are sensitive to alpha particles are used extensively in commercial radon detectors.

Chemical etching of the material takes place on all surfaces that are exposed to the etching solution. The exposed surfaces of the material are eroded along with the material along the track. Therefore, the rate of etching has to be carefully controlled to get the maximum amount of information from the track. Notice that etching of a uniform track will generally form a circular cone because the material will be more easily removed from the surface than from deep along the track. Mica tracks are diamond-shaped due to the lattice structure as opposed to being circular.

Nuclear emulsions are closely related "track detectors" that trace their origins to the original discovery of radiation by Becquerel. Nuclear emulsions are very fine-grained photographic film. The film is "exposed" by the passage of radiation through it and the grains of AgC1 are activated by the ionization. The film is developed and with careful handling and microscopic observation, the track or path of individual particles can be traced. Occasionally, a particle interacts with a nucleus in the emulsion, creating many fragments or particles, and the tracks of the reaction products can be traced. The emulsion

-35-

is also sensitive to the rate of ionization and the nature of the particle in each track can often be determined. On the other hand, most people are familiar with the shadow images of skeletal features taken with x-rays. The x-rays are absorbed and scattered more efficiently by the heavy elements in bones (essentially calcium) than by the light elements in soft tissue (carbon, oxygen, hydrogen) and create a shadow. The grains in the emulsion are then exposed by the transmitted x-rays and are developed to form the negative image.

#### 18.4 Nuclear Electronics and Data Collection

As we have seen, essentially all of the nuclear radiation detectors produce electronic pulses in response to the interaction of some ionizing radiation. These signals are processed by standardized *nuclear instrumentation modules* (NIM) electronics to count the number of pulses or to more fully analyze the size or even the shape of the signal. In addition, computer-based electronics in the CAMAC (Computer Automated Measurement and Control) system are used to measure the time relationships of pulses, the pulse heights, and the signal shapes. The signals are recorded and stored by computers for later analysis. An important feature of scientific studies with radioactivities and with nuclear beams is that the data must be collected as rapidly as possible usually during a very limited time. A radioactive source will decay away after being produced and cannot be "stopped" because the scientist is not ready to use it. Similarly, the nuclear reactions induced by particle beams take place in a very short time and must be recorded when they occur. Then after a set of events has been collected "on-line," the data are analyzed "off-line."

We will give a very brief overview of the kinds of modules used. CAMAC and NIM electronics fall into three broad categories, *linear electronics* that maintain a linear

relationship to the size of the initial signal, *logic circuits* that provide only a standard (or single sized) pulse indicating that a given logical condition was met, and *data-acquisition modules* to measure the signals and record the data. One should realize that with modern high density electronics the functions that we will describe can correspond to a single electronic module or may be condensed into a single integrated circuit. Therefore, we will only describe the *functions* performed by the electronic modules and not specific equipment.

The output of most detectors is an electrical pulse that carries information about the energy deposited in the detector, the time of the interaction, etc. Linear electronics are described as modules that preserve and extract information about the energy deposit in the detector from the detector signal. An overview of these modules and their function is given in Table 18-1. A typical pulse-height analysis system is shown in Figure 18-21. The signal from the detector is given a preliminary amplification and shaping by a *preamp* before being sent through a coaxial cable to a *linear amplifier*. This is done to prevent noise in the cable from destroying the tiny detector signal. In the amplifier the signal is further amplified and shaped before analysis. The height of the pulse is related to the energy deposited in the detector. The analog-to-digital converter (ADC) converts the signal from the amplifier into digital data (a number of standard pulses) thus measuring its size. The ADC could be contained on a plug-in card in a personal computer (used to measure the distribution of pulses from a single detector monitoring a radioactive source) or it might be one of many identical ADC units in a CAMAC module (used to record the signals from many detectors monitoring nuclear collisions simultaneously)

-37-

Logic modules are used to monitor the counting rate of single detectors and the relative times at which radiation is detected. A fast signal derived from the detector itself, the preamplifier, or from a timing-filter amplifier is sent to a *discriminator*. The discriminator produces an output pulse with a fixed shape (generally square) and size when the input signal crosses a reference. Discriminators usually have multiple identical output signals. The logic pulses can be sent to a scaler that simply counts the number of pulses, to a count rate meter to monitor radiation rates or doses, and to a time-to-amplitude converter (TAC) to measure the relative times of arrival of two or more logic signals.

#### **18.5** Nuclear Statistics

Radioactive decay is a random process. The number of nuclei in a sample of radioactive material that decay in any time period is not a fixed number but will differ, usually, for various time periods. This point can be readily shown by making repeated measurements of the activity of a long-lived radionuclide, each for the same time duration. The results of such an experiment might be as shown in Table 18-2. Note that in these measurements there is a large range of activity values with a clustering near the center of the range. We can plot these data (Table 18-2) as a distribution function, by "binning" the data (Figure 18-22). We can now ask ourselves if we can understand this distribution function. Statisticians have given us mathematical models that describe these and other similar distribution functions. As a background for our discussion of how to extract the maximum amount of information from these data, let us consider some of these models.

The most general model to describe radioactive decay is the *binomial distribution.* For a process that has two outcomes (success or failure, decay or no decay), we can write for the distribution function P(x)

$$P(x) = (n!/((n-x)!x!)p^{x}(1-p)^{n-x}$$
(18-9)

where n is the number of trials where each trial has a probability of success p and P(x) is the predicted probability of getting x successes. Applying this distribution to radioactivity, P(x) might be taken as the probability of getting x counts in a given time interval and  $p=\lambda\Delta t$ where  $\Delta t$  is a time short compared with the measurement time and the half-life. Note that x and n are both integers. Typical binomial distribution functions are shown in Figure 18-23.

The binomial distribution function is cumbersome and a simplification can be made. If the probability of success p is small (p<<1) (the measurement time is very short compared with the half-life), we can approximate the binomial distribution by the Poisson distribution. The Poisson distribution is written as

$$P(x) = (x_m)^x \exp(-x_m)/x!$$
(18-10)

where

$$x_{\rm m} = {\rm pn}$$
 (18-11)

Thus we have a simplified distribution characterized by one parameter,  $x_m$  compared to two parameters in the binomial distribution. The Poisson distribution is an asymmetric distribution as shown in Figure 18-24. Besides being a more tractable function to use, the Poisson distribution has certain important properties that we will use in analyzing radioactivity data. Let us consider a parameter, the *variance*,  $\sigma^2$ , which expresses something about the width of the distribution of values about the mean,  $x_m$ . For a set of N measurements, we can calculate  $\sigma^2$  as

$$\sigma^{2} = \frac{\sum_{i=1}^{N} (x_{i} - x_{m})^{2}}{N - 1}$$
(18-12)

(For the data of Table 18-2,  $x_m$ =1898,  $\sigma$ =44.2). For a binomial distribution

$$\sigma^2 = np(1-p)$$
 (18-13)

which is cumbersome to use. But, for a Poisson distribution, we can show that

$$\sigma^2 = \mathbf{x}_{\mathrm{m}} \tag{18-14}$$

$$\sigma = (x_m)^{1/2}$$
(18-15)

Applying these equations to the data of Table 18-2, we get  $\sigma_{Pois}$  = 43.6. This illustrates the important point that these distribution functions are models, not physical laws, and when they are applied to finite data sets, their predictions may deviate from observation.

The Poisson distribution can be applied also to describe the action of detectors. For example, suppose the interaction of a  $\gamma$ -ray photon with an inefficient scintillator produced, on average, 3.3 photoelectrons from the photocathode. The probability of producing no photoelectrons (not seeing the event) is given by the Poisson distribution as

$$P(0) = \exp(-3.3) = 3.7\% \tag{18-16}$$

Thus 3.7% of the events will be missed due to "statistical fluctuations".

A further simplification of the parent binomial distribution occurs when the number of successes is relatively large, *i.e.*, we get more than about 30 counts in a measurement. Then the binomial distribution can be represented as a *normal* or *Gaussian distribution*. Here we write

$$P(x) = \frac{1}{\sqrt{2\pi x_m}} \exp\left(-\frac{(x - x_m)^2}{2x_m}\right)$$
(18-17)

This analytical approximation is symmetric (Figure 18-25). As shown in Figure 18-25, 68.3% of the measured values lie within  $\pm 1\sigma$  of the mean,  $x_m$ . Furthermore 95.5% of all measurements lie within  $\pm 2\sigma$  of the mean and 99.7% lie within  $\pm 3\sigma$  of the mean. The full width at half maximum (FWHM) is 2.35 $\sigma$ .

Thus for a single measurement of a count rate of 100, we would estimate that  $\sigma = 10$ . We could say, with a 68.3% chance of being correct that the true rate was between 100-10=90 and 100+10=110. With 95.5% certainty, we could say the true rate lies between 80 and 120. Generalizing, we can quote the results of a measurement as x " n $\sigma$  where n is related to the probability that an infinite number of measurements would give a value within the quoted range. For n = 0.6745,1,1.6449,1.96,2,2.5758,3, the "confidence limits" are 50%, 68.3%, 90%,95%,95.5%, 99%, and 99.7%, respectively. Commonly people will quote the results of a measurement as x ±  $\sigma$ . One should remember that doing so means one will be wrong 31.7% of the time, *i.e.*, the mean count rate will be outside x ± $\sigma$ . If this risk is not acceptable, one should pick a greater confidence level, *i.e.*, 2 $\sigma$ , 3 $\sigma$ , etc.

Another distribution function of interest relates to the distribution of time intervals between successive counts. We know the average time between counts is (1/count rate). The distribution of time intervals is given by the *interval distribution*. This distribution (applicable to all random events) states that for a process with an average time between events t<sub>m</sub>, the probability of getting a time t between successive events is

$$I(t) = \frac{1}{t_m} \exp(-t/t_m) dt$$

(18-18)

For radioactive decay

$$t_{\rm m} = 1/\lambda \tag{18-19}$$

This distribution function is shown in Figure 18-26. Note the most probable time between events is zero. Random events (counts, natural disasters, etc.) occur in "bunches."

Let us summarize how we describe the statistical uncertainty in measurements of radioactivity. If we measure the activity of a sample (+background) as 64 counts in 1 minute, then we estimate

$$(S+B) = 64 \text{ cpm}$$

with an uncertainty  $\sigma_{S+B}$ 

$$\sigma_{S+B} = 8 \text{ cpm}$$

What if a second measurement with no sample showed a background of 10 counts in 1 minute? We would then estimate

B = 10 cpm  
$$\sigma_{\rm B} = (10)^{1/2} = 3.2$$
 cpm

What is the net sample rate and its uncertainty? This raises the general question of calculating the uncertainty in the result of some mathematical operations on an uncertain number. If we consider two independently determined numbers and their uncertainties (standard deviations),  $A \pm \sigma_{a}$ ,  $B \pm \sigma_{b}$ , we can write down, as shown in Table 18-3, some rules for the uncertainty in the result of some common mathematical operations.

As an example of the use of the relations outlined in Table 18-3, we would calculate that for our sample and background counting case,

Net rate = (sample + background) - (background)

Uncertainty in net rate =  $(8^2 + 3.2^2)^{1/2} = 8.6$  cpm

Up to now we have carefully restricted our discussion of nuclear statistics to cases where l-min counts were taken. If the number of counts recorded in 1 min was x, then the counting *rate* has been quoted as  $x \pm (x)^{1/2}$  cpm. Suppose, however, that we recorded 160 counts in 5 min. What would be the standard deviation of the average counting rate (in cpm)? The best estimate of the mean number of counts in the 5-min period would by 160 ± (160)<sup>1/2</sup> that is, 160 ± 13 counts. The average rate would be 160/5 ± 13/5 = 32 ± 3 cpm. In general, therefore, the rate R is given as

R=(number of counts recorded)/(measurement time)=x/t

The standard deviation of the rate,  $\sigma_R$ , is

$$\sigma_{\rm R} = (x)^{1/2} / t = (R^* t)^{1/2} / t = (R/t)^{1/2}$$
(18-20)

Thus for the preceding example we could have calculated directly that

$$\sigma_{\rm R} = ({\rm R}/t)^{1/2} = (32/5)^{1/2} = 3$$

Often we wish to compute the average of two numbers,  $x_1$ , and  $x_2$ , both of which have an uncertainty denoted by their standard deviations  $\sigma_1$  and  $\sigma_2$ , respectively. The best average of these two numbers is not the simple average but weighted average  $x_m$ , given by

$$x_{m} = \left(\frac{x_{1}}{\sigma_{1}^{2}} + \frac{x_{2}}{\sigma_{2}^{2}}\right) / \left(\frac{1}{\sigma_{1}^{2}} + \frac{1}{\sigma_{2}^{2}}\right)$$

$$x_m = \frac{x_1 + wx_2}{1 + w}$$

where

$$w = \left(\frac{\sigma_1}{\sigma_2}\right)^2$$

In short, each number is weighted by the inverse of its standard deviation squared. For the weighted average of N values,  $x_i$ , with standard deviation,  $\sigma_i$ , we have

$$x_m = \frac{\sum_{i=1}^{N} \left(\frac{x_i}{\sigma_i^2}\right)}{\sum_{i=1}^{N} \left(\frac{1}{\sigma_i^2}\right)}$$

The uncertainty or standard deviation of x is given by

$$\sigma_{x_m} = \left(\frac{1}{\sum_{i=1}^{N} \left(\frac{1}{\sigma_i^2}\right)}\right)^{1/2}$$

For example, suppose that we make two independent measurements of an activity, obtaining results of  $35 \pm 10$  cpm and  $46 \pm 2$  cpm. The weighted average of the two measurements is

w = 
$$(10/2)^2 = 25$$
  
x<sub>m</sub> =  $((35)(1+(25)(46))/(1+25) \approx 46$  cpm

The standard deviation of the weighted average is

$$\sigma_x = ((100+(25)^2(4))/(26^2))^{1/2} \approx 2.0$$

Thus we would say that the average rate was  $46 \pm 2$  cpm.

#### 18.5.1 Rejection of Abnormal Data

In our discussions so far, we have only considered the uncertainty in the experimental data due to the randomness of radioactive decay. But there may also be systematic error that contributes to the overall uncertainty in the data. As a result, when we make repeated measurements of a sample activity under seemingly identical situations, we will find occasionally one measurement that differs from the others by a large amount. If included in the average, this abnormal observation may cause significant error. When are we justified in rejecting such data? One criterion for rejecting such data is to reject suspected values that deviate from the mean by more than  $2\sigma$  or  $3\sigma$ . The probabilities of occurrence of such deviations are 4.5 and 0.27%, respectively.

What about the question of whether a detector or counting system is working properly? For example, the data in Table 18-2 do not exactly match a Poisson or normal distribution. Was the counting system malfunctioning? One parameter that we can calculate that will help us answer such questions is  $\chi^2$  (chi-squared). Formally

$$\chi^{2} = \frac{\sum_{i=1}^{N} (x_{i} - x_{m})^{2}}{x_{m}}$$

For the data in Table 18-2, we calculate that  $\chi^2 = 37194/1898 = 19.60$ . Figure 18-27 shows the properties of the  $\chi^2$  distribution in terms of p, the probability that a random sample of N values from a Poisson distribution. For the data of Table 18-2,  $\chi^2 / v = 1.03$ , which is acceptable. To be suspicious of the data  $\chi^2 / v$  should have been >1.7 or <0.6.

#### 18.5.2 Setting Upper Limits When No Counts Are Observed

Suppose your experiment failed to detect a type of decay you were seeking: What can you say about its occurrence? The simplest answer is what is termed the *"one-event upper limit"*. You assume that you had detected one event, and calculate the resulting decay rate, cross section, etc., taking into account detection efficiencies, solid angles, etc.

A more sophisticated answer can be obtained by using the properties of a Poisson distribution. The probability of observing n events if the mean value is  $\mu$  is given as

$$\rho(n/\mu) = \frac{\mu^n}{n!} = e^{-\mu}$$

The probability of observing 0 events in a time period T for a process with mean rate  $\lambda$  is

$$\rho(0/\lambda T) = e^{-\lambda T}$$

It can be shown that the upper limit on the rate (when zero counts are observed),  $\lambda_0$ , is given by

$$\lambda_0 = -\frac{1}{T}\ln(1 - CL)$$

where CL is the confidence limit you wish to attach to your upper limit. (If you want to quote an upper limit with 95% confidence, then CL = 0.95.)

#### Example

Suppose you are attempting to observe a process that should produce 1 decay per week and you observe no counts (with a 100% efficient detector) in 4 weeks. At the 95% confidence level, the upper limit on the decay rate is

$$\lambda_0 = -\frac{1}{4}\ln(1 - 0.95) = 0.75 / week$$

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#### **Problems**

- What detector would you chose to detect the following? Please give your reasoning. (a) 0.1 μCi <sup>32</sup>P? (b) a mixture of 0.1 μCi <sup>3</sup>H and 0.5 μCi <sup>14</sup>C (c) a mixture of 10 nCi each of <sup>235</sup>U and <sup>238</sup>U. (d) 50 Bq <sup>60</sup>Co.
- An advertisement for a high purity Ge detector quoted its relative efficiency as being 200%. Is that possible? Please explain.
- 3. How would you detect 10 MeV neutrons?
- 4. How many counts would you have to collect to have 90% confidence in the quoted counting rate?
- 5. How would you detect the individual  $\beta$ -particles,  $\gamma$ -rays and delayed neutrons from a fission product mixture?

- 6. What is the maximum allowed counting rate with a scintillation detector with a 0.25  $\mu$ s dead time if you can only tolerate a 3% counting loss?
- Given five measurements of the counting rate of a long-lived sample that are 2712, 2753, 2708, 2739 and 2726. Do a chi-squared test to see if these data are consistent with a Poisson distribution.
- 8. How many counts do you need to collect to have a 2% uncertainty in your result?
- Sketch a diagram of the nuclear instruments you would need to make logical AND, OR, NAND and NOR circuits.
- 10. If a sample has an average counting rate of 16 cts/hr and you read out the data every hour for each of 1000 hours, how many times will you get 2, 8, 16 and 32 counts?



Fig. 18-1. Schematic representation of a parallel-plate ionization chamber in which one ion pair has been formed. V is the voltage source, R denotes resistor and C denotes the capacitor. From O'Kelley.



**Fig. 18-2.** Schematic diagram of a gridded ionization chamber. From O'Kelley



**Figure 18-3.** The schematic diagram of an ion chamber that drifts the ionization perpendicular to the particle's path is shown. In this case the anode is segmented and the relative rate of ionization along the path can be determined. The device also contains a Frisch grid between the anode and chamber to improve the pulse-shape response of the device. (c) The schematic version of a detector that drifts the ionization along the particle's path, called a Bragg counter, is shown. The time distribution of the output signal will contain information on the relative rate of ionization all along the particle's path.



**Fig. 18-4.** Schematic illustration of the Townsend avalanche in a gas ionization device. (From Knoll). The avalanche occurs very close to the wire in reality.



**Figure 18-5.** (a) Diagram of a  $2\pi$  gas flow proportional counter. The sample is introduced by sliding out the bottom of the chamber. (b) a  $4\pi$  gas flow proportional counter for absolute counting.



**Figure 18-6.** Schematic block diagram of the components of a proportional

counter.



**Figure 18-7.** Characteristic curve for a proportional counter.



Figure 18-8. Schematic view of the crystal lattice of Si. The dots represent

electron pair bonds between the Si atoms.



**Figure 18-9.** A schematic diagram of the energy levels of crystalline silicon.



**Figure 18-10.** Schematic diagram of a typical n-type impurity in a silicon crystal lattice



**Figure 18-11.** A schematic diagram of the energy levels of crystalline silicon with a donor impurity.



Figure 18-12. A schematic diagram of a silicon crystal lattice with a p--type

impurity in it.



**Figure 18-13.** Schematic diagram of the energy levels of silicon with a p-type impurity.



**Figure 18-14.** Schematic diagram of a silicon crystal lattice with a p-type impurity boron, at one lattice point. The hole is labeled 0, while the electrons are denoted as 1,2, etc.



**Figure 18-15.** Schematic diagram of a p-n junction



Figure 18-16. Representation of Silicon diodes, simple SSB and segmented.



Figure 18-17. The  $\gamma$ -ray spectrum of neutron-activated lung tissue as measured with a 3 x 3" NaI(Tl) detector and a Ge detector. From Cooper.



**Figure 18-18.** A schematic view of the scintillation mechanism in organic crystals.



**Figure 18-19.** The electronic energy levels in an ionic crystal.



Fig. 18-20. (a) Cutaway diagram of a typical solid scintillation detector. (b)
Diagram of a scintillation detector illustrating schematically the way
in which light from the scintillator is transmitted to a photomultiplier
tube. A typical wiring diagram is shown for the 10-stage
photomultiplier operated with a positive high-voltage supply. From
O'Kelley.



**Figure 18-21.** Schematic diagram of a simple pulse height analysis system for nuclear spectroscopy.



**Figure 18-22.** Distribution of counts of a long-lived sample.



Figure 18-23. Binomial distributions for various values of N and p. (From Leo).



Figure 18-24. Poisson distribution for various values of  $x_m$ . (From Leo.)



Figure 18-25. A plot of the normal distribution function showing the mean  $x_m$  and

 $1\sigma$  and  $2\sigma$  points.



Figure 18-26. A plot of the frequency of occurrence of a time interval t between

successive counts, where the average rate is 1 cpm.



Figure 18-27. A plot of the chi-squared distribution

Tables

Linear Input- Linear Output	Input	Output
Preamplifier	Linear charge pulse from detector	Linear Tail Pulse
Linear Amplifier	Linear tail pulse	Amplified and shaped linear pulse
Biased Amplifier	Shaped linear pulse	Linear pulse proportional tp amplitude of input pulse that lies above input bias level
Pulse Stretcher	Fast linear pulse	Conventional shaped linear pulse of amplitude equal to input pulse
Sum Amplifier	Two or more shaped linear pulses	Shaped linear pulse with amplitude equal to the sum of the coincident input pulses
Delay	Fast linear or shaped linear pulse	Identical pulse after a fixed time delay
Linear Gate	(1)Shaped linear pulse (2) Gate pulse	Linear pulse identical to linear input if gate pulse is supplied in time

# Table 18-1.Summary of Common Pulse-Processing Modules

		overlap
Linear Input- Logic Output	Input	Output
Integral Discriminator	Shaped linear pulse	Logic pulse if input amplitude exceeds discriminator level
Single Channel Analyzer	Shaped linear pulse	Logic pulse if input amplitude lies within acceptance window
Time Pickoff	Fast linear or shaped linear pulse	Logic pulse synchronized with some feature of input pulse
Logic Input – Linear Output	Input	Output
Time to Amplitude Converter	Logic start and stop pulses separated by a time ∆t	Shaped linear pulse with amplitude proportional to At
Logic Input- Logic Output	Input	Output
Coinidence	Logic pulses at two or more inputs	Logic pulse if pulses appear at all inputs within a time interval $\Delta t$
Anti- Coincidence	Logic pulses at two inputs	Logic pulse only if pulse appears at one input without pulse at second input

		within time Δt
Scaler	Logic Pulses	One logic pulse for N input pulses

# Table 18-2 Typical Sequence of Counts of a long-Lived Sample (<sup>170</sup>Tm)\*

Measurement Number	cp0.1m	Xi- X <sub>m</sub>	(xi- Xm) <sup>2</sup>
1	1880	-18	324
2	1887	-11	121
3	1915	17	289
4	1851	-47	2209
5	1874	-24	576
6	1853	-45	2025
7	1931	33	10899
8	1886	-32	1024
9	1980	82	6724
10	1893	-5	25
11	1976	78	6084
12	1876	-22	484
13	1901	3	9
14	1979	81	6561
15	1836	-62	3844
16	1832	-66	4536

17	1930	32	1024
18	1917	19	361
19	1899	1	1
20	1890	-8	64

\*We are indebted to Prof. R.A. Schmitt for providing these data.

### Table 18-3. Uncertainties for some common operations

Operation	Answer	Uncertainty
Addition	A+B	$(\sigma_A^2 + \sigma_B^2)^{1/2}$
Subtraction	A-B	$(\sigma_A^2 + \sigma_B^2)^{1/2}$
Multiplication	A*B	$A^*B((\sigma_A/A)^2+(\sigma_B/B)^2)^{1/2}$
Division	A/B	$A/B((\sigma_A/A)^2+(\sigma_B/B)^2)^{1/2}$