Translational partition function

Thermodynamic quantities

Average energy

$$\overline{E} = kT^2 \left(\frac{\partial \ln q}{\partial T}\right)_{V,N}$$

as $q = \frac{V}{\lambda_a^3} \Rightarrow q = \frac{V}{\left(\frac{h}{2\pi m kT}\right)^{3/2}}$

$$\Rightarrow q = V \times \left(\frac{2\pi m kT}{h}\right)^{3/2}$$

$$\Rightarrow \ln q = \ln V + \frac{3}{2}\ln\left(\frac{2\pi m kT}{h}\right) + \frac{3}{2}\ln T$$

$$\left(\frac{\partial \ln q}{\partial T}\right)_{V,N} = \frac{3}{2T}$$

Hence average energy

$$\overline{E} = kT^2 \frac{3}{2T}$$

$$\boxed{\overline{E} = \frac{3}{2}kT}$$

Total energy ; $\boxed{E = \frac{3}{2}NkT}$

Total molar internal energy

$$U = \frac{3}{2} N_A KT = \frac{3}{2} RT$$

Heat Capacity

$$\overline{C_V} = \left(\frac{\partial U}{\partial T}\right)_V = \frac{3}{2} R$$

Helmholtz free energy

$$A = -NKT \ln Q$$

$$= -NKT \left[\ln V + \frac{3}{2} \ln\left(\frac{2\pi m KT}{h^2}\right)\right]$$

Now, pressure

$$P = -\left(\frac{\partial A}{\partial V}\right)_{T,N}$$

$$= +NKT \frac{\partial}{\partial V}\left[\ln V + 3/2 \ln\left(\frac{2\pi m KT}{h^2}\right)\right]$$

$$\boxed{P = \frac{NKT}{V}}$$