

**Tutorial-6, Statistical Mechanics & Others (Paper-203), January 20,2016**

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**Maximum Marks: 50**

Q-1. Rewrite  $Z(V, T, \mu) = \sum_{N=0}^{\infty} \lambda^N Z(N, V, T)$  in terms of population ( $n_k$ ) of individual quantum states ( $\varepsilon_k$ ) where  $\lambda = e^{\mu/k_B T}$ . Work out-  $S = \sum_{N=0}^{\infty} \sum_{\{n_k\}}^* x_1^{n_1} x_2^{n_2} \dots \sim S = \prod_{K=1}^2 (1 + x_K + x_K^2 \dots)$ .

Q-2. Prove that  $\bar{p}V = k_B T \ln(Z(V, T, \mu))$  of a grand-canonical ensemble. What are thermodynamic characteristic functions of different ensembles?

Q-3. Obtain quantum statistics of *Fermi-Dirac* ('+') gas ( $Z_{FD}(V, T, \mu)$ ) using Pauli's *Exclusion Principle* (Restricted occupancy). Give examples of some fermions.

Q-4. Obtain quantum statistics of *Bose-Einstein* (-) gas ( $Z_{BE}(V, T, \mu)$ ) (Unrestricted occupancy). Give some examples of bosons.

Q-5. Find internal energy ( $\bar{E}$ ) of *Fermi-* and *Bose-* gases.

Q-6. How is chemical potential ( $\mu$ ) of quantum gases modified at classical limits-  $N/V \rightarrow 0$  and  $T \rightarrow \infty$ ?

Q-7. Find  $\bar{N}$ , average occupancy of individual quantum states ( $\bar{n}_k$ ) and  $p - V$  relations for *Fermi-Dirac* and *Bose-Einstein* gases.

Q-8. Obtain classical limits of  $\bar{E}$  and  $p - V$  for *F-D* and *B-E* gases.

Q-9. Discuss similarities and dissimilarities of photons and phonons.

Q-10. Find quantum statistics of a gas with *maximum occupancy* 'm'. (Gentile's theory)

Q-11. Why in rotational spectra, the ground state ( $J=0$ ) is not mostly populated? Find  $J_{max}$  in terms of temperature ( $T$ ) and rotational temperature constant ( $\theta_r$ ).

Q-12. What are the basic postulates of Einstein's theory of specific heat of monoatomic crystal? Compare  $C_v$  (Dulong and Petit's) in light of Einstein's theory of  $C_v$  for monoatomic crystal at high temperature ( $T$ ).

Q-13. Predict  $C_v$  at  $T \rightarrow 0$  by Einstein's theory on monoatomic crystal.

Q-14. Find equilibrium constant  $K_c$  in terms of partition functions of reactants and products. Obtain relation between  $K_p$  and  $K_c$ .

Q-15. What is the ratio of  $J_{max}$  of  $H_2$ ,  $HD$  and  $D_2$  in rotational spectra?

Books: McQuarrie (Statistical Mechanics), Callen (Thermodynamics and Thermostatistics), Nash (Elements of Statistical Thermodynamics), Atkins (Physical Chemistry), Landau &

Lifshitz (Statistical Physics),MC Gupta (Statistical Mechanics).