$$S_1 = \sum_{n_x=1}^{\infty} e^{-\alpha_1 n_x^2}$$

$$\simeq \int_0^{\infty} e^{-\alpha_1 x^2}\, dx$$ + some small corrections that can be neglected when $\frac{\lambda_d}{a} << 1$.

$$\simeq \int_0^{\infty} e^{-z} \times \frac{1}{\sqrt{\alpha_1}} \frac{1}{2} z^{-1/2}\, dz$$

Let $\alpha_1 x^2 = z$

$x^2 = \frac{z}{\alpha_1} \Rightarrow x = \sqrt{\frac{z}{\alpha_1}}$

$dx = \frac{1}{\sqrt{\alpha_1}} \frac{1}{2} z^{-1/2}\, dz$

$$\simeq \frac{1}{2\sqrt{\alpha_1}} \int_0^{\infty} e^{-z} z^{-1/2}\, dz$$

$$\simeq \frac{1}{2\sqrt{\alpha_1}} \times \sqrt{\frac{1}{2}}$$

$$\simeq \frac{\sqrt{\pi}}{2\sqrt{\alpha_1}} \Rightarrow \frac{1}{2} \frac{\sqrt{\pi \times 8mKTa^2}}{\sqrt{h^2}}$$ (putting the value of α_1)

$$\simeq \frac{\sqrt{2\pi m KT} \times a}{h} \Rightarrow \frac{a}{\lambda_a}$$

where $\lambda_a = \frac{h}{\sqrt{2\pi m KT}}$ called thermal debroglie wavelength

Similarly $S_2 = \frac{b}{\lambda_d}$

$S_3 = \frac{c}{\lambda_d}$

Hence single particle partition function

$$Q_{total} = S_1 S_2 S_3$$

$$= \frac{abc}{\lambda_d^3}$$

$$\boxed{Q_{total} = \frac{V}{\lambda_d^3}} \longrightarrow$$ ~~Transl~~ Translational partition function.