TOPIC: Entropy of Monoatomic gases

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$$S = k_B \ln(Z(N,V,T)) + k_B T\left(\frac{\partial \ln Z(N,V,T)}{\partial T}\right)_{N,V}$$

for

Canonical ensemble of Monoatomic gases

$$\text{OR, } TS = E - A$$

$$\Rightarrow S = -\frac{A}{T} + \frac{E}{T}$$

Already derived: $E \equiv$ Internal Energy, $A \equiv$ Helmholtz Free energy

Thus,

$$S = Nk_B\left(\ln\left[\left(\frac{2\pi m k_B T}{h^2}\right)^{\frac{3}{2}}\left(\frac{Ve}{N}\right)\right] + \ln\left(\omega_{e1} + \omega_{e2}e^{-\beta\Delta\varepsilon_{12}}\right)\right) + \frac{3}{2}Nk_B + \left(\frac{Nk_B\,\omega_{e2}\,\beta\Delta\varepsilon_{12}\,e^{-\beta\Delta\varepsilon_{12}}}{q_{elec}}\right)$$

Sackur-Tetrode Equation

$$S = Nk_B\left(\ln\left[\left(\frac{2\pi m k_B T}{h^2}\right)^{\frac{3}{2}}\left(\frac{Ve}{N}\right)\right] + \frac{3}{2}\right) + Nk_B\left(\ln\left(\omega_{e1} + \omega_{e2}e^{-\beta\Delta\varepsilon_{12}}\right) + \left(\frac{\omega_{e2}\,\beta\Delta\varepsilon_{12}\,e^{-\beta\Delta\varepsilon_{12}}}{q_{elec}}\right)\right)$$

$$\Rightarrow \boxed{S = Nk_B\left(\ln\left[\left(\frac{2\pi m k_B T}{h^2}\right)^{\frac{3}{2}}\left(\frac{Ve^{5/2}}{N}\right)\right]\right) + S_{elec}}$$

GIBB'S PARADOX ON ENTROPY: Brain-Teaser**

- Removal of Partition causes mixing | Distinguishability leads to paradox ??
- Entropy should be doubled being extensive prop.

$V = V_1 + V_2$

N_1 Particles, V_1 = Volume, S_1 = Entropy	N_2 particles, V_2 = Volume, S_2 = Entropy

Partition

TOPIC :- Chemical Potential of Monoatomic gases

$$\boxed{\mu = -k_B T \left(\frac{\partial \ln(Z(N,V,T))}{\partial N}\right)_{V,T}}$$

We know $Z(N,V,T) = \dfrac{(q_{trans}\, q_{elec}\, q_{nucl})^N}{N!}$

$$\Rightarrow \ln(Z(N,V,T)) = N \ln(q_{trans}\, q_{elec}\, q_{nucl}) - (N \ln(N) - N)$$

$$= N\left(\ln(q_{tran}\, q_{elec}\, q_{nucl}) + \ln\left(\frac{e}{N}\right)\right)$$

$$\Rightarrow \left(\frac{\partial \ln(Z(N,V,T))}{\partial N}\right)_{V,T} = \ln(q_{tran}\, q_{elec}\, q_{nucl}) - \left(\left(N \cdot \frac{1}{N}\right) + \ln(N) - 1\right)$$

$$= \ln\left((q_{trans}\, q_{elec}\, q_{nucl})/N\right)$$

Rewriting,

$q_e = q_{elec}$

$q_n = q_{nucl}$

$$\Rightarrow \mu = -k_B T \ln\left[\left(\frac{2\pi m k_B T}{h^2}\right)^{\frac{3}{2}} \left(\frac{V}{N}\right)\right] - k_B T \ln(q_e\, q_n)$$

{Since $V = \dfrac{N k_B T}{p}$ For ideal gases}

$$\Rightarrow \boxed{\mu = \underbrace{-k_B T \left(\ln\left[\left(\frac{2\pi m k_B T}{h^2}\right)^{\frac{3}{2}} k_B T\right] - \ln(q_e q_n)\right)}_{\mu_0(T)} + k_B T \ln(p)}$$

or $\boxed{\mu = \mu_0(T) + k_B T \ln(p)} \rightarrow$ Classical Thermodynamic Eq.