TOPIC :
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q_{trans} By other method (Density of states)

No. of energy states b/w ε and $\varepsilon + d\varepsilon$ (In Next Appendix)

$$\omega(\varepsilon)d\varepsilon = \frac{\pi}{4}\left(\frac{8ma^2}{h^2}\right)^{3/2}\varepsilon^{1/2}\,d\varepsilon$$

⇒ Total number of states constitutes Partition function : (as ε spans "0" to "∞")

$$q_{trans} = \int_0^\infty \omega(\varepsilon)\,e^{-\beta\varepsilon}\,d\varepsilon$$

$$= \int_0^\infty \frac{\pi}{4}\left(\frac{8ma^2}{h^2}\right)^{3/2}\varepsilon^{\frac{1}{2}}e^{-\beta\varepsilon}\,d\varepsilon$$

Substituting $\varepsilon = x^2$
⇒ $\varepsilon^{\frac{1}{2}}d\varepsilon = 2x^2dx$

$$= \frac{\pi}{4}\left(\frac{8ma^2}{h^2}\right)^{3/2}\int_0^\infty \varepsilon^{\frac{1}{2}}e^{-\beta\varepsilon}\,d\varepsilon$$

Standard Integral

$$\int_0^\infty x^{2n}e^{-ax^2}dx = \frac{1.3..(2n-1)}{2^n\,a^n}\left(\frac{\pi}{a}\right)^{\frac{1}{2}}$$

$$= \frac{\pi}{4}\left(\frac{8ma^2}{h^2}\right)^{\frac{3}{2}}\times 2\int_0^\infty x^2e^{-\beta x^2}dx$$

$$= \frac{\pi}{2}\left(\frac{8ma^2}{h^2}\right)^{\frac{3}{2}}\left(\frac{1}{2\beta}\right)\left(\frac{\pi}{\beta}\right)^{\frac{1}{2}}$$

Rearranging,

$$= \left(\frac{1}{4}\right)\left(\frac{8ma^2}{h^2}\right)^{\frac{3}{2}}\left(\frac{\pi}{\beta}\right)^{3/2}$$

$$= \left(\frac{2\pi mk_BT}{h^2}\right)^{3/2}V$$

CONCEPT OF THERMAL DE-BROGLIE WAVELENGTH

$$q_{trans} = \left(\frac{2\pi m k_B T}{h^2}\right)^{3/2} V$$

Points to be noted : $q_{trans} \propto V$

$\Rightarrow q_{trans} \equiv$ linear function of Volume, V

$$\Rightarrow q_{trans} = V \Big/ \left(\frac{h^2}{2\pi m k_B T}\right)^{3/2}$$

$$= V / \Lambda^3$$

where $\Lambda = \left(h^2 / 2\pi m k_B T\right)^{\frac{1}{2}}$

$\downarrow$

Thermal De-Broglie wavelength

- $\langle p^2 \rangle / 2m = \frac{3}{2} k_B T$ ($\because$ Equipartition Energy)

$\Rightarrow p \propto (m k_B T)^{1/2}$

$\Rightarrow \Lambda = \left(h^2 / 2\pi m k_B T\right)^{\frac{1}{2}} \propto \left(\frac{h}{p}\right)$

$\Rightarrow \Lambda \propto (h/p) \Rightarrow \Lambda \propto \underset{\text{De-Broglie W. L.}}{\lambda}$

Thus $\Lambda \Rightarrow$ Thermal De-Broglie Wave Length

$\Rightarrow (\Lambda^3 / V) \lll 1 \Rightarrow$ Thermal de-Broglie Wavelength must be very small compared to size of container.